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Constraining the lateral dimensions of uniaxially loaded materials increases the calculated strength and stiffness: application to muscle and bone

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If a solid body is deformed along one direction, by a uniaxial applied stress for instance, then strains will also be induced in perpendicular directions. The negative ratio of the induced strain to the applied strain is known as the Poisson ratio. Analysis of the elasticity tensor relating stress and strain within a solid shows that if the induced strain is restricted, then a greater stress is required to produce the same strain; it appears stiffer. Many biological materials with a mechanical function are subject to forces which are primarily uniaxial. This mechanism appears to be used to maximize the uniaxial load-bearing properties of some of these materials. Muscles are commonly surrounded by strong sheets of connective tissue which will constrain the lateral expansion of the muscle as it contracts. This increases the stress in the muscle for a given strain, and hence the load it can support. Similarly, cancellous bone is normally surrounded by a shell of much stronger compact bone and this effectively increases the stiffness of the cancellous bone without the penalty of increasing the mass, as would be the case if the same stiffening was produced by increasing the degree of calcification. It also has important implications for the failure of bone, which is largely a function of strain rather than stress.

1. Introduction

If a solid body is deformed along one direction, by a longitudinally applied stress for instance, then deformations are also induced in the lateral directions perpendicular to this. These deformations are related to the primary longitudinal deformation by the Poisson ratios appropriate for that material. For an isotropic solid there is only one Poisson ratio, and that has to lie in the range -1.0 to $+0.5$, the latter representing an isovolumetric deformation. For materials belonging to other symmetry groups there are up to six Poisson ratios, although these are not all independent [1]. Poisson ratios are defined for a body subject only to a uniaxial stress, all other directions in the body being unconstrained. If, however, the lateral dimensions of the body are somehow constrained and the changes in dimension induced in them by the uniaxial stress are restricted, then this will affect the apparent mechanical properties of the material pertinent to the applied stress. This effect is examined by analysing the general elasticity tensor relating strain to stress within the material. The results are applied to muscle and bone, where this effect could be important to the mechanical function of these tissues.

Skeletal muscles are made up of muscle fibres held together in various configurations by a connective tissue framework. These muscles are themselves surrounded by sheets of connective tissue termed the epimysium [2, 3] or, more generally, fascia [4]. Various functions have been proposed for the intramuscular connective tissue, such as organizing the components of the muscle and transmitting forces within the muscle [2, 5, 6]. However, the function of the epimysium, which surrounds each muscle, and of the numerous fasciae, which enclose many groups of muscles, is much less well understood [4]. These fasciae may be very substantial; for instance, the fascia lata which envelops the thigh [7] and the thoracolumbar fascia surrounding the erector spinae muscles, the mechanical function of which has not yet been satisfactorily explained [8]. The very existence of such sheets of connective tissue surrounding some of the most powerful muscle groups in the body suggests that they have a strategic mechanical function.

Cancellous, or trabecular, bone is found in the centre of the vertebral bodies, the femoral head and in many other bones in the vertebrate skeleton. It consists of a lattice-like structure of trabeculae whose

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geometrical arrangement depends on the geometry and load system associated with it. It is always at least partially surrounded by a thin shell of compact, or cortical, bone which is much stronger and stiffer. Isolated cancellous bone is relatively weak. Recent testing of cancellous bone has shown that its mechanical properties when tested *in situ* are enhanced considerably over those when the surrounding cancellous bone is removed to make a standard testpiece [9]. Cortical bone would be expected to have a similar effect. The interplay between the two types of bone therefore has important consequences for the mechanical strength of the whole tissue.

2. Theory

The strains, ε_{ij}, in a linearly elastic material corresponding to stresses, σ_{kl}, are related by the compliance tensor s_{ijkl}, using standard summation nomenclature over the subscripts [10]. In its most general form this has 36 independent components, but this number is often reduced when the symmetry of the material is considered. This relationship may be expressed in matrix form which reduces the number of subscripts needed to describe the components [10]

$$\varepsilon_i = s_{ij}\sigma_j \qquad (1)$$

where ε_i are the strains produced in the material by the applied stresses σ_j, and s_{ij} are the components of the compliance tensor written in matrix form [10]. This assumes that the material properties are linear elastic and ignores any time-dependence. These assumptions may not be true for muscle or connective tissue but should not affect the overall conclusions presented here providing, as seems reasonable, the stress is a monotonically increasing function of strain and the elastic limit is not exceeded.

Choosing the 3-axis along which to apply a uniaxial stress, σ_3, then all of the other σ_j are zero and the resultant strains are

$$\varepsilon_3 = s_{33}\sigma_3 \qquad (2)$$

$$\varepsilon_i = s_{i3}\sigma_3 \qquad (3)$$

Expressions for the Poisson ratios can then be found from their definition using Equations 2 and 3:

$$\nu_{3i} = -\varepsilon_i/\varepsilon_3 = -s_{i3}/s_{33} = -s_{3i}/s_{33} \qquad (4)$$

noting that $s_{ij} = s_{ji}$ [10].

If now stresses σ_1 and σ_2, of the same sign as σ_3, are applied to restrict the strains induced in these directions and represent the effects of a constraint then to achieve the same strain ε_3 as before a new stress σ_3' is required:

$$\varepsilon_3 = s_{31}\sigma_1 + s_{32}\sigma_2 + s_{33}\sigma_3' \qquad (5)$$

$$\varepsilon_i = s_{i1}\sigma_1 + s_{i2}\sigma_2 + s_{i3}\sigma_3' \qquad (6)$$

However, ε_3 is given by Equation 2, and substituting this in Equation 5 gives

$$s_{33}\sigma_3 = s_{31}\sigma_1 + s_{32}\sigma_2 + s_{33}\sigma_3' \qquad (7)$$

and therefore

$$\sigma_3' = \sigma_3 - (s_{31}\sigma_1 + s_{32}\sigma_2)/s_{33} \qquad (8)$$

and from Equation 4

$$\sigma_3' = \sigma_3 + (\nu_{31}\sigma_1 + \nu_{32}\sigma_2) \qquad (9)$$

Special cases may be derived from this relationship. If the applied constraining stress is cylindrically symmetrical, for instance a hoop or band around a cylindrical body, then $\sigma_1 = \sigma_2 = \sigma_r$, say, and Equation 9 becomes

$$\sigma_3' = \sigma_3 + (\nu_{31} + \nu_{32})\sigma_r \qquad (10)$$

If the material itself also has cylindrical symmetry in its mechanical properties then the Poisson ratios become degenerate and may be written as $\nu_{31} = \nu_{32} = \nu_{3r}$, say, and

$$\sigma_3' = \sigma_3 + 2\nu_{3r}\sigma_r \qquad (11)$$

For an isotropic material there is only one Poisson ratio and a similar expression to Equation 11 is obtained with $\nu_{3r} = \nu$. The above analysis is perfectly general and is not dependent on the direction of the stress σ_3, the important point is that the constraining stresses in a perpendicular direction must be of the same sign as the longitudinal stress so as to oppose the strains being induced. Only systems in which the state of stress is compressive are considered here.

Applying a lateral constraint to a material subject to a uniaxial longitudinal stress will therefore increase the stress needed to produce the same longitudinal strain compared with when it is unconstrained. The increase depends on the Poisson ratios as well as the lateral stresses applied. Stiffness is defined as the ratio of stress to strain and, as applying a radial constraint increases the axial stress, it similarly increases the apparent axial stiffness. As the strain is defined to be the same in both of the cases derived above, the strength and the stiffness are both increased by the same fractional amount, f, where

$$f = (\sigma_3' - \sigma_3)/\sigma_3 \qquad (12)$$

and the appropriate expression can be found by substituting for $(\sigma_3' - \sigma_3)$ from one of Equations 9 to 11. A similar strengthening of a material was noted by Filon [11] who showed that discrepancies between various measurements of the strength and stiffness of stone, considered to be isotropic, depended on whether the blocks being tested were allowed to expand laterally as they were compressed. Higher values of strength and stiffness were obtained if the lateral expansion of the loaded faces of the block was restricted, and his analysis showed that this was a consequence of the constraint.

3. Muscle

3.1. Fascia

The so-called deep fasciae surrounding many muscles are sheets of connective tissue composed chiefly of collagen fibres arranged with a high degree of regularity; the fibres in one layer usually being at an angle to those in the next layer [4]. Collagen is very strong in tension, although weak in compression or flexion, and when stretched provides a restoring force to oppose the applied stress [12]. The organization of collagen fibres in a tissue largely determines its extensibility; for instance, the ligamentum flavum of the spine is much more extensible than the posterior longitudinal ligament and has a much less ordered arrangement of collagen fibres in the unstretched position, the fibres become aligned as the tissue is stretched [13, 14]. The collagen fibres in the annulus fibrosus of the intervertebral disc are arranged in layers, called lamellae, and the fibre direction in adjacent lamellae alternates between $+65°$ and $-65°$ to the spinal axis [15]. This structure enables the intervertebral disc to function as a pressure vessel in which internal pressure in the nucleus pulposus is resisted by tension in the annulus fibrosus [16]. A comparable structure has been demonstrated in the thoracolumbar fascia of the human spine, with fibre angles of $\pm 60°$ to the spinal axis [8]. A similar mechanism may therefore apply to the epimysium and fascia where, instead of a pressure, the radial strain generated by the contraction of a muscle will tend to stretch the surrounding epimysium and fascia. The restoring force generated in the connective tissue will then apply a radially directed stress to the muscle.

The thoracolumbar fascia is anchored at its lateral and medial margins in such a way that the angle of $\pm 60°$ between the collagen fibres and the longitudinal axis of the erector spinae muscles does not change much during flexion of the spine, being $\pm 50°$ in the fully flexed spine [8]. It can be shown that the fibre angle has to be $> 54.7°$ for the fibres to be able to apply any restraint to the lateral expansion [16, 17]. In this case the fascia is capable of affecting the apparent muscle strength and stiffness throughout a significant part of the range of movement. This need not be the case for other fasciae. If the fascia or epimysium is attached to the muscle in a similar way to that which has been described for the perimysium [18], then the fibre angle will change considerably over the range of contraction of the muscle and may only have a strengthening effect over the last stages.

3.2. Isovolumetric contraction

Many skeletal muscles act directly between two points of attachment and consist of a large number of muscle fibres running parallel to their length, each of which will be rotated through some arbitrary angle with respect to those surrounding it. Muscles may therefore be considered to have cylindrical symmetry about an axis defined along the muscle between the points of attachment so Equation 11 is the relevant one to use in this case. Because muscle is isovolumetric [19], a lower bound on the value of ν_{3r} may be found by considering the contraction of an unconstrained cylindrical muscle from an initial length l_0 and radius r_0 to a new length l_1 and radius r_1. Then

$$(r_1/r_0)^2 = l_0/l_1 \qquad (13)$$

From the definition of engineering strain

$$l_1 = l_0(1 + \varepsilon_3) \quad \text{and} \quad r_1 = r_0(1 + \varepsilon_r) \qquad (14)$$

and Equation 13 then becomes

$$(1 + \varepsilon_r)^2 = 1/(1 + \varepsilon_3) \qquad (15)$$

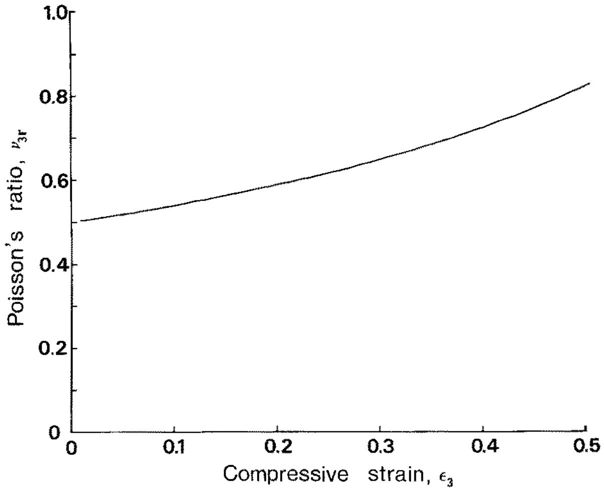

Figure 1 Variation of the Poisson ratio, ν_{3r}, with compressive strain, ε_3, for an isovolumetric contraction of a cylindrically symmetric material, such as muscle, to half its original length.

and hence

$$\varepsilon_r = (1 + \varepsilon_3)^{-1/2} - 1 \qquad (16)$$

Combining Equation 16 with Equation 4 then yields the following expression for the Poisson ratio:

$$\nu_{3r} = -[(1 + \varepsilon_3)^{-1/2} - 1]/\varepsilon_3 \qquad (17)$$

This function tends to a minimum value of 0.5 as ε_3 tends to zero and is shown in Fig. 1 for values of compressive strain up to 0.5, i.e. $\varepsilon_3 = -0.5$, the muscle contracting to half its original length. This implies, from Equation 11, that the longitudinal stress is increased by at least the amount of the radial stress applied.

3.3. State of stress and its magnitude

It is important to note that both the longitudinal stress and strain in the muscle, caused by the muscle contracting, are compressive and therefore take a negative sign. Muscle is unusual as a material, since stresses are generated internally whereas mechanical properties are generally related to stresses applied externally. Therefore, although any external load exerts a tensile force on the muscle and a muscle is commonly described as being "in tension", the actual internal state of stress must be considered to be compressive so as to oppose the external load and contract the muscle. If this were not so and the muscle was in a state of tensile stress, this would act in co-operation with the external load and the muscle would extend catastrophically. The radial stress generated by stretching the fascia, being an externally applied compressive stress, also takes a negative sign.

An estimate of the order of magnitude of the radial constraining stress, σ_r, generated by a fascia surrounding a muscle may be found by considering, for example, the thoracolumbar fascia of the spine. This fascia forms a cylindrical sheath surrounding the erector spinae muscles. If it functions in a similar way to a pressure vessel, the circumferential stress, σ_c, can be related to the internal pressure or, in this case, radial stress, σ_r. Adapting the standard formula for such a vessel [20] then gives

$$\sigma_r = \sigma_c t/r \qquad (18)$$

where t is the thickness of the fascia and r the radius of the cylinder that it forms. The thickness of the thoracolumbar fascia in a human is estimated to be about 1 mm and the radius has been measured from magnetic resonance images to be approximately 50 mm [21]. An estimate of the maximum safe working stress that the fascia could sustain may be obtained by comparison with the surface zone of articular cartilage in which the safe maximum stress is calculated to be of the order of 10 MPa and which contains an internal pressure of about 300 kPa [22]. Putting $\sigma_c = 10^7$ Pa, $t = 10^{-3}$ m and $r = 0.05$ m in Equation 18 suggests that σ_r would be about 200 kPa. The actual working stress in the surface zone of articular cartilage has recently been calculated to be about 1 MPa [23] and using this value for σ_c in Equaton 18 results in a radial stress of about 20 kPa. The radial stress calculated above cannot be equated with the intracompartmental pressure found in various muscle compartments [24]. This pressure is an isotropic fluid pressure and is not the same as the stress in the solid phase of the material generated by expansion against the constraint of the fascia. The solid phase will also prevent the occlusion of blood vessels that would otherwise occur if the muscle was simply pressurized.

Estimates of the maximum active strength of muscle vary considerably, but typical values seem to be about 200 to 400 kPa, measured in isolated skeletal muscles from animals [25] and humans [26]. Substituting these values for σ_3 yields a range of values for the fractional increase in strength and stiffness, f (from Equations 11 and 12), of between 0.5 and 1.0 using the higher value from above for the stress in the fascia and taking the Poisson ratio to be 0.5. If the stress in the fascia is lower, about 1 MPa, then the fractional increase is between 0.05 and 0.1. Constraining muscular expansion by surrounding it with a strong connective tissue fascia may therefore increase the strength and stiffness of the muscle by about 5 to 10%, but possibly by as much as 100%. The volume of connective tissue is small relative to that of muscle, about 4% in the above example. This would suggest that the energy needed to stretch the fascia is only a small fraction of that used to move the external load. However, because of the relatively high stiffness of connective tissue, the stresses produced could have a significant effect on the muscle stress. Connective tissues are also viscoelastic, which means that their stiffness increases as the rate of straining increases. Thus, rapid contraction of a muscle would be expected to generate a greater strengthening and stiffening effect by the fascia. Increasing the stiffness of a muscle will also increase the bending stiffness, and this may be important for increasing the dynamic stability of the flexible spine [27].

Muscle strengths measured [28] or calculated [29] *in situ* in the human body are reported to be in the range 400 to 1000 kPa, a curious discrepancy from those measured *in vitro*. Removal of the fascia lata from the thigh has been reported to result in muscle herniation in 50% of patients after 3.5 years and a reduced strength in hip flexion and knee extension of about 7% compared with the non-operated leg [30]. A hypothesis that explains these observations is provided by the analysis presented here, which suggests that the surrounding connnective tissue may significantly increase muscle strength.

4. Bone

Cancellous bone has a measured average stiffness of about 0.3 GPa and a density of about 240 kg m^{-3} and is usually found in the skeleton largely surrounded by compact bone, which has a stiffness of about 20 GPa and a density of about 2000 kg m^{-3}. From this it can be seen that using cancellous bone rather than compact bone, where mechanical considerations allow, results in a considerable saving in mass. Forces in the long bones of the skeleton and in the spine are predominantly uniaxial, arising from body weight and muscular forces which commonly have a large component along the bone to which they are attached. The preceding analysis shows that in this case the surrounding of cancellous bone with a shell of compact bone can enhance the mechanical properties of the cancellous bone by increasing its apparent stiffness. The mechanical failure of bone in *in vitro* tests is strongly correlated with the strain applied rather than the stress [31, 32] and increasing the stiffness would therefore be expected to result in a similar increase in strength.

Recent mechanical tests on cancellous bone from the upper tibial epiphysis from human knees have demonstrated just such an effect [9]. Compressive tests to 0.8% strain were performed *in situ* on a slice of cancellous bone larger than the ends of the compression platens. A cylindrical plug was subsequently machined out from the same place as tested *in situ*, and used as the standard, and finally this plug was encased by a sliding-fit steel jacket to constrain lateral expansion. It was found that the *in situ* test and the constrained test produced values for the stiffness at 0.8% strain of 19 and 22% more, respectively, than the standard testpiece.

This is a significant stiffening of the cancellous bone, and shows that care must be exercised when extrapolating results from isolated mechanical tests to the *in vivo* structure. A similar stiffening effect might be expected from the shell of cortical bone which surrounds most cancellous bone, especially as loads are generally distributed over the whole cross-section and are not simply a localized load as in the above tests. It also confirms that the cortical bone has a strategic function in strengthening the cancellous bone and does not simply have secondary functions, such as a containment structure for the bone marrow, for instance, as suggested for vertebrae on the basis of finite-element modelling [33].

5. Conclusions

The following conclusions can be drawn.

1. It is shown that constraining the lateral expansion, or the Poisson ratio effects, in a body subject to a uniaxial stress increases the apparent strength and stiffness of the material.

2. Applied to muscle, this leads to the hypothesis that the tough sheets of connective tissue commonly

surrounding muscles will increase the force that a muscle can apply for a given strain and, as a consequence, increase its apparent stiffness.

3. Applied to bone, it explains recent results showing that isolated cancellous bone appears weaker and less stiff than when it is tested surrounded by other bone. On the basis of the theory, this result is extrapolated to suggest a similar effect from surrounding cancellous bone with compact bone. Using these two types of bone together would therefore be expected to produce a material that is light and strong.

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